37

MTP-ASTR-G-63-7) CTS:
July 9, 1963

N64 11229 *

CODE-1
(NASA-TMX-51204)

33p.

GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

MULTI-SPEED RESOLVERS FOR ANALOG DIGITAL CONVERSION OF SHAFT ANGLES

By

H. E. Thomason

[redacted]

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MSFC - Form 523 (Rev. November 1960)

35.

GEORGE C. MARSHALL SPACE FLIGHT CENTER

MTP-ASTR-G-63-7

MULTI-SPEED RESOLVERS FOR ANALOG DIGITAL CONVERSION OF SHAFT ANGLES

By

H. E. Thomason

ABSTRACT

11229

The problems associated with accurately measuring platform gimbal angles digitally with use of a multi-speed resolver are discussed. Electrically, the system is quite unsophisticated and depends strictly on old and well-known techniques. The division of one revolution (2π) into exactly n/2 discrete parts, where n is the number of poles, is a purely digital aspect of the two-speed resolver. If the number of poles could be very large, the system could be considered that of an incremental encoder; then the detailed structure of the fine signal between zeros could be ignored. But, the number of poles is limited by the number of slots that can be put into a lamination of a given size. Therefore, the critical area of the system design is in the front end, where the resolver phase clock in combination with zero detectors transforms the input angle θ into a pulse time. System accuracy is gained or lost here.

Basically, the front end consists of the two-speed resolver, plus a few resistors and capacitors which compose the phase clock. A pair of counters and a high-fidelity excitation amplifier complete the system.

AUTHOR

GEORGE C. MARSHALL SPACE FLIGHT CENTER

MTP-ASTR-G-63-7

MULTI-SPEED RESOLVERS FOR ANALOG DIGITAL CONVERSION OF SHAFT ANGLES

By

H. E. Thomason

ASTRIONICS DIVISION

LIST OF ILLUSTRATIONS

gear passes. The two-speed resolver considered for platform angle measurements is an electrical device which, through a multiplicity of poles and a method winding, produces the same course and fine signals as a gear coupled pair.

The basic conversion system will consist of a chain of the following three transformations.

1. Input angle to carrier voltage amplitudes (function of resolver)

2. Voltage amplitudes to carrier phase time (function of networks)

3. Phase time to carrier synchronous clock count (function of digital counters)

It is observed that the measurement is analog up to the point where the count is taken. The multiple-speed resolver is the link connecting sources (angles) to sinks and storages in a generalized network. This analysis will first investigate the influence of the network on resolver performance. Then the entire network connected as a phase clock is analyzed to determine source and magnitude of errors in the overall angle to phase time transformation.

DISCUSSION

The 32:1 multiple-speed resolver schematic is shown in FIGURE 1 where one winding of the rotor is excited, and the other rotor quadrature winding is short-circuited.

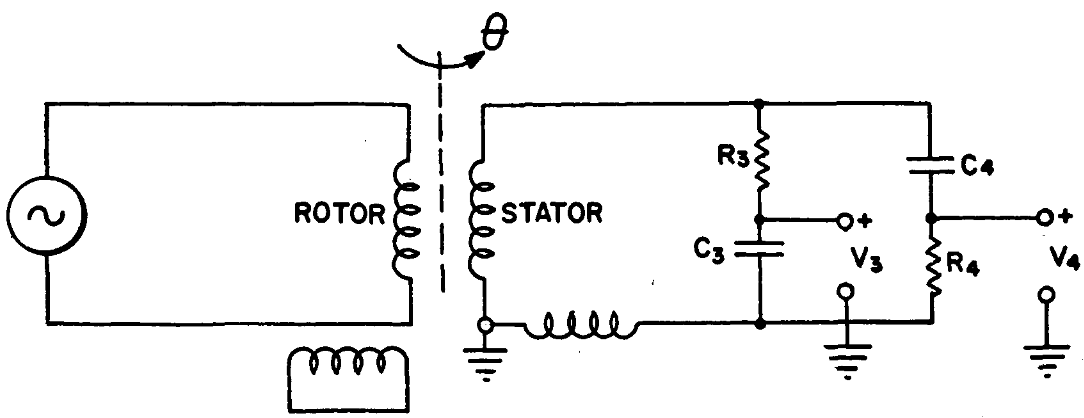


FIGURE 1. MULTIPLE-SPEED RESOLVER SCHEMATIC

GEORGE C. MARSHALL SPACE FLIGHT CENTER

MTP-ASTR-G-63-7

MULTI-SPEED RESOLVERS FOR ANALOG DIGITAL CONVERSION OF SHAFT ANGLES

By

H. E. Thomason

SUMMARY

The problems associated with accurately measuring platform gimbal angles digitally with use of a multi-speed resolver are discussed. Electrically, the system is quite unsophisticated and depends strictly on old and well-known techniques. The division of one revolution (2π) into exactly $n/2$ discrete parts, where n is the number of poles, is a purely digital aspect of the two-speed resolver. If the number of poles could be very large, the system could be considered that of an incremental encoder; then the detailed structure of the fine signal between zeros could be ignored. But, the number of poles is limited by the number of slots that can be put into a lamination of a given size. Therefore, the critical area of the system design is in the front end, where the resolver phase clock in combination with zero detectors transforms the input angle θ into a pulse time. System accuracy is gained or lost here.

Basically, the front end consists of the two-speed resolver, plus a few resistors and capacitors which compose the phase clock. A pair of counters and a high-fidelity excitation amplifier complete the system.

INTRODUCTION

Multiple-speed resolvers are being used as digital shaft encoders for analog-to-digital conversion of the platform gimbal angles. Originally, the term "multiple-speed resolvers" denoted resolvers coupled by integral ratio

The two stator windings are tied in series and loaded with resistor condenser elements as illustrated in FIGURE 1. At the excitation frequency, $R_3 C_3 \omega_3 = R_4 C_4 \omega_4$ and $R_3 = R_4$.

The two voltages V_3 and V_4 are used to generate a start and stop pulse, respectively, where $V_3 = 0; \frac{dV_3}{dt} > 0$ and $V_4 = 0; \frac{dV_4}{dt} > 0$. The start pulse ($V_3$) is to open the gating circuit to a counter and allow a fixed high-frequency time base to be counted until a stop pulse is generated by V_4. The number of pulses counted is a measure of the shaft rotation of the resolver.

To develop the principle of operation, first consider an ideal resolver encoder as shown in FIGURE 2.

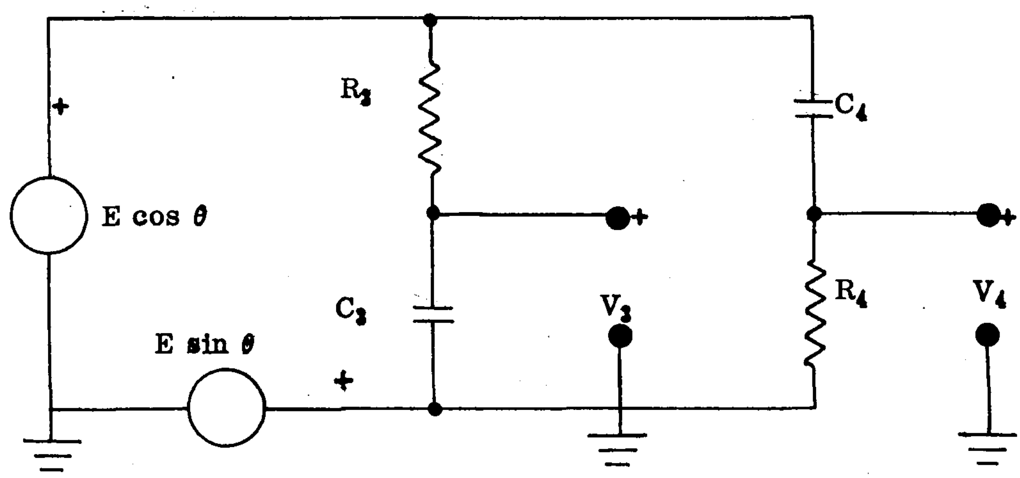


FIGURE 2. IDEAL RESOLVER ENCODER NETWORK

The voltages V_3 and V_4 can be expressed as:

$$V_3 = \frac{1}{R_3 C_3 S + 1} (E \cos \theta - E \sin \theta) + E \sin \theta \tag{1}$$

$$V_4 = \frac{R_4 C_4 S}{R_4 C_4 S + 1} (E \cos \theta - E \sin \theta) + E \sin \theta \tag{2}$$

At the excitation frequency, let

$$R_3 C_3 \omega = R_4 C_4 \omega = 1 \tag{3}$$

then by substitutes and trigonometric identities:

$$V_3 = \frac{E}{\sqrt{2}} \quad e^{j(\theta - \frac{\pi}{4})} \tag{4}$$

$$V_4 = \frac{E}{\sqrt{2}} \quad e^{-j(\theta - \frac{\pi}{4})} \tag{5}$$

The phase angle of V_3 relative to V_4 is obtained from equations 4 and 5.

$$\frac{V_3}{V_4} = e^{j(2\theta - \frac{\pi}{2})} \tag{6}$$

It is seen from equation 6 that the phase angle between voltages V_3 and V_4 is a two-cycle function of angular rotation.

The instantaneous voltages V_3, V_4, and the resolver open circuit stator voltage can be expressed as:

$$V_3\ (t) = \frac{E}{\sqrt{2}} \sin \left[\omega t + (\theta - \frac{\pi}{4}) \right]$$

$$V_4\ (t) = \frac{E}{\sqrt{2}} \sin \left[\omega t - (\theta - \frac{\pi}{4}) \right]$$

$$V_{oc}\ (t) = E \sin \omega\, t \tag{7}$$

Thus at $\theta = 0$, V_4 (t) leads V_{oc} (t) by 45 degrees as illustrated in FIGURE 3.

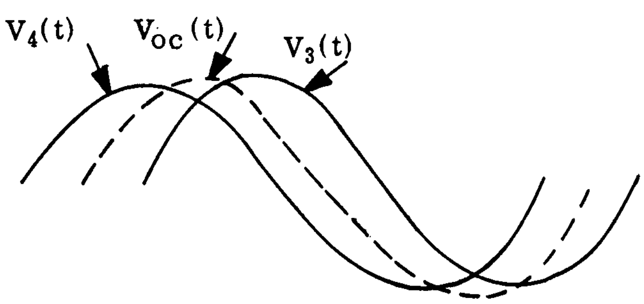


FIGURE 3. OUTPUT VOLTAGE REFERENCED TO OPEN CIRCUIT VOLTAGE

At zero phase shift, the shaft angle θ must be 45 degrees. Therefore, the single-speed winding zero setting will have to be shifted 1.40625 degrees from the zero position of the 32 speed winding. As θ increases in a positive direction from 45 degrees, V_3 will lead the open circuit stator voltage and V_4 will lag the open circuit stator voltage.

An actual resolver encoder network is described in FIGURE 4 where no error will be considered in the sin θ and cos θ terms.

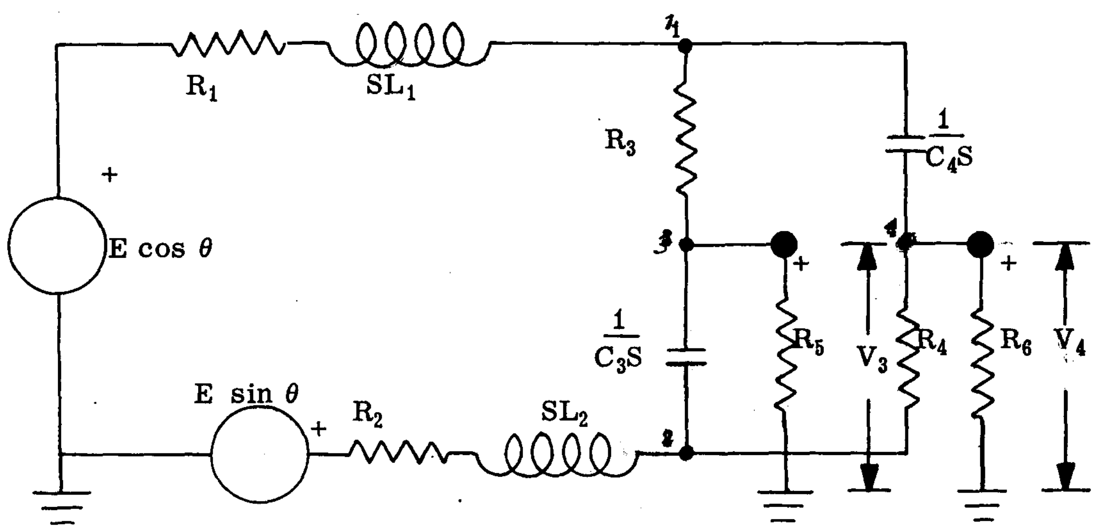


FIGURE 4. ACTUAL RESOLVER ENCODER NETWORK

The resolver source impedances $(R_2 + SL_2)$, $(R_1 + SL_1)$, and the load resistances R_5 and R_6 will cause the actual encoder to differ from the ideal encoder shown in FIGURE 2. A two-cycle error term in the phase angle between voltages V_3 and V_4 will be caused by the source impedances and the load resistance.

Redraw FIGURE 4 as FIGURE 5, and write the describing equations based on a nodal analysis.

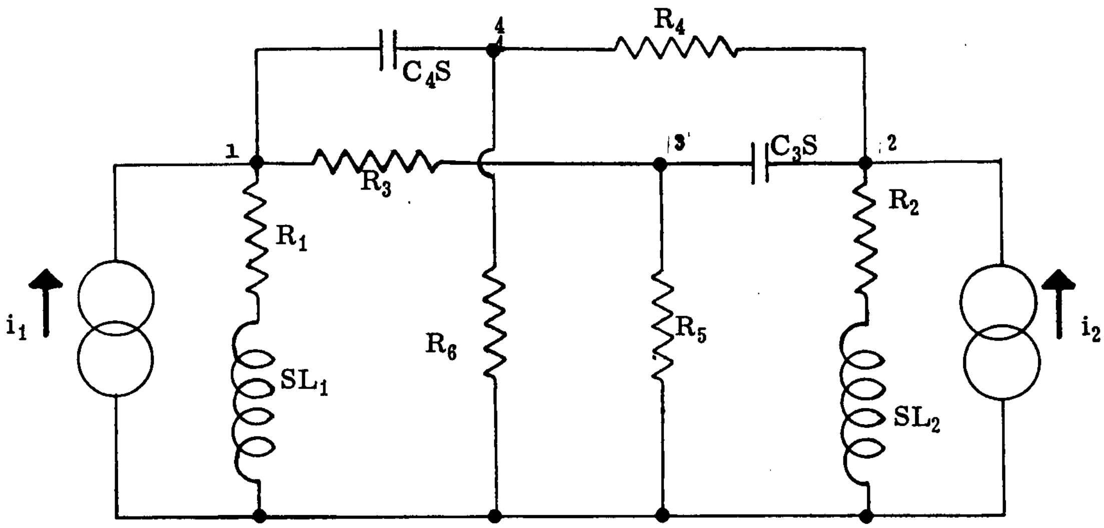


FIGURE 5. NODAL SCHEMATIC

$$i_1 = \frac{E\cos\theta}{(R_1 + SL_1)} \qquad \text{and } i_2 = \frac{E\sin\theta}{(R_2 + SL_2)}$$

$$i_1 = \left(\frac{1}{R_1 + SL_1} + \frac{1}{R_3} + C_4S\right) V_1 + \left(\frac{-1}{R_3}\right) V_3 + 0\,V_2 + (-C_4S)\,V_4$$

$$0 = \left(\frac{-1}{R_3}\right) V_1 + \left(\frac{1}{R_5} + \frac{1}{R_3} + C_3S\right) V_3 + (-C_3S)\,V_2 + 0\,V_4$$

$$i_2 = 0\,V_1 + (-C_3S)\,V_3 + \left(C_3S + \frac{1}{R_4} + \frac{1}{R_2 + SL_2}\right) V_2 + \left(\frac{-1}{R_4}\right) V_4$$

$$0 = (-C_4S)\,V_1 + 0\,V_3 + \left(\frac{-1}{R_4}\right) V_3 + \left(C_4S + \frac{1}{R_4} + \frac{1}{R_6}\right) V_4$$

(8)

The network equations in matrix form are:

$$\begin{bmatrix} i_1 \\ 0 \\ i_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{R_1+SL_1} + \frac{1}{R_3} + C_4S\right) & \frac{-1}{R_3} & 0 & -C_4S \\ \frac{-1}{R_3} & \left(\frac{1}{R_5} + \frac{1}{R_3} + C_3S\right) & -C_3S & 0 \\ 0 & -C_3S & \left(C_3S + \frac{1}{R_4} + \frac{1}{R_2+SL_2}\right) & \frac{-1}{R_4} \\ -C_4S & 0 & \frac{-1}{R_4} & \left(C_4S + \frac{1}{R_4} + \frac{1}{R_6}\right) \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_3 \\ V_2 \\ V_4 \end{bmatrix}$$

The admittance matrix or system determinant is defined as Δ. Minus signs are omitted since only the amplitudes are important. Then V_3 and V_4 can be formulated:

$$V_3 = \frac{1}{\Delta}\left\{ i_1 \begin{bmatrix} \frac{-1}{R_3} & -C_3S & 0 \\ 0 & \left(C_3S + \frac{1}{R_4} + \frac{1}{R_2+SL_2}\right) & \frac{-1}{R_4} \\ -C_4S & \frac{-1}{R_4} & \left(C_4S + \frac{1}{R_4} + \frac{1}{R_6}\right) \end{bmatrix} + i_2 \begin{bmatrix} \left(\frac{1}{R_1+SL_1} + \frac{1}{R_3} + C_4S\right) & 0 & -C_4S \\ \frac{-1}{R_3} & -C_3S & 0 \\ -C_4S & \frac{-1}{R_4} & \left(C_4S + \frac{1}{R_4} + \frac{1}{R_6}\right) \end{bmatrix} \right\} \tag{9}$$

$$V_4 = \frac{1}{\Delta}\left\{ i_1 \begin{bmatrix} \frac{-1}{R_3} & 0 & -C_4S \\ \left(\frac{1}{R_5}+\frac{1}{R_3}+C_3S\right) & -C_3S & 0 \\ -C_3S & \left(C_3S+\frac{1}{R_4}+\frac{1}{R_2+SL_2}\right) & \frac{-1}{R_4} \end{bmatrix} \right.$$

$$\left. + i_2 \begin{bmatrix} \left(\frac{1}{R_1+SL_1}+\frac{1}{R_3}+C_4S\right) & \frac{-1}{R_3} & -C_4S \\ \frac{-1}{R_3} & \left(\frac{1}{R_5}+\frac{1}{R_3}+C_3S\right) & 0 \\ 0 & -C_3S & \frac{-1}{R_4} \end{bmatrix} \right\} \qquad (10)$$

which can be expanded into the following equations:

$$\Delta V_3 = i_1 \left\{ \frac{1}{R_2\left[(R_2)^2 - (SL_2)^2\right]} \left[R_2\left(\frac{1}{R_4}+\frac{1}{R_6}\right) - S^2 L_2 C_4\right] \right.$$

$$\left. + \frac{1}{R_3 R_4 R_6} + C_3 C_4 S^2 \left[\frac{1}{R_3}+\frac{1}{R_4}\right] \right\}$$

$$+ i_1 S \left\{ \frac{1}{R_3\left[(R_2)^2 - (SL_2)^2\right]} \left[-L_2\left(\frac{1}{R_4}+\frac{1}{R_6}\right) + R_2 C_4\right] \right.$$

$$\left. + \frac{C_3}{R_3}\left(\frac{1}{R_4}+\frac{1}{R_6}\right) + \frac{C_4}{R_3 R_4} \right\}$$

$$+ i_2 \left\{ \frac{S^2 C_3}{\left[(R_1)^2 - (SL_1)^2\right]} \left[-L_1 \left(\frac{1}{R_4} + \frac{1}{R_6}\right) + R_1 C_4 \right] \right.$$

$$\left. + C_3 C_4 S^2 \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6} \right] \right\}$$

$$+ i_2 S \left\{ \frac{C_3}{\left[(R_1)^2 - (SL_1)^2\right]} \left[R_1 \left(\frac{1}{R_4} + \frac{1}{R_6}\right) - S^2 L_1 C_4 \right] \right.$$

$$\left. + \frac{C_3}{R_3} \left[\frac{1}{R_4} + \frac{1}{R_6} \right] + \frac{C_4}{R_3 R_4} \right\} . \tag{11}$$

$$\Delta V_4 = i_1 \left\{ \frac{C_4 S^2}{\left[(R_2)^2 - (SL_2)^2\right]} \left[R_2 C_3 - L_2 \left(\frac{1}{R_3} + \frac{1}{R_5}\right) \right] \right.$$

$$\left. + \left[C_3 C_4 S^2 \right] \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \right] \right\}$$

$$+ i_1 S \left\{ \frac{C_4}{\left[(R_2)^2 - (SL_2)^2\right]} \left[R_2 \left(\frac{1}{R_3} + \frac{1}{R_5}\right) - S^2 L_2 C_3 \right] \right.$$

$$\left. + \frac{C_4}{R_4} \left[\frac{1}{R_3} + \frac{1}{R_5} \right] + \frac{C_3}{R_3 R_4} \right\}$$

$$+ i_2 \left\{ \frac{1}{R_4 \left[(R_1)^2 - (SL_1)^2\right]} \left[R_1 \left(\frac{1}{R_3} + \frac{1}{R_5}\right) - S^2 L_1 C_3 \right] \right.$$

$$\left. + \frac{1}{R_3 R_4 R_5} + C_3 C_4 S^2 \left[\frac{1}{R_3} + \frac{1}{R_4} \right] \right\}$$

$$+ i_2 S \left\{ \frac{1}{R_4 \left[(R_1)^2 - (SL_1)^2 \right]} \left[R_1 C_3 - L_1 \left(\frac{1}{R_3} + \frac{1}{R_5} \right) \right] \right.$$

$$\left. + \frac{C_4}{R_4} \left[\frac{1}{R_3} + \frac{1}{R_5} \right] + \frac{C_3}{R_3 R_4} \right\} \tag{12}$$

To simplify equations 11 and 12, make the following substitution where $S = j\omega$ (steady state)

$$A = \frac{1}{R_3 \left[(R_2)^2 + (\omega L_2)^2 \right]} \left[R_2 \left(\frac{1}{R_4} + \frac{1}{R_6} \right) + \omega^2 L_2 C_4 \right] + \frac{1}{R_3 R_4 R_6}$$

$$- C_3 C_4 \omega^2 \left[\frac{1}{R_3} + \frac{1}{R_4} \right] \tag{13}$$

$$B = \frac{-\omega^2}{\left[(R_1)^2 + (\omega L_1)^2 \right]} \left[-L_1 \left(\frac{1}{R_4} + \frac{1}{R_6} \right) + R_1 C_4 \right]$$

$$- C_3 C_4 \omega^2 \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6} \right] \tag{14}$$

$$C = \frac{\omega}{R_3 \left[(R_2)^2 + (\omega L_2)^2 \right]} \left[-L_2 \left(\frac{1}{R_4} + \frac{1}{R_6} \right) + R_2 C_4 \right]$$

$$+ \frac{\omega C_3}{R_3} \left[\frac{1}{R_4} + \frac{1}{R_6} \right] + \frac{C_4 \omega}{R_3 R_4} \tag{15}$$

$$D = \frac{\omega C_2}{\left[(R_1)^2 + (\omega L_1)^2 \right]} \left[R_1 \left(\frac{1}{R_4} + \frac{1}{R_6} \right) + \omega^2 L_1 C_4 \right]$$

$$+ \frac{C_3 \omega}{R_3} \left[\frac{1}{R_4} + \frac{1}{R_6} \right] + \frac{C_4 \omega}{R_3 R_4} \tag{16}$$

$$K = \frac{-C_4\omega^2}{\left[(R_2)^2 + (\omega L_2)^2\right]} \left[R_2 C_3 - L_2 \left(\frac{1}{R_3} + \frac{1}{R_5}\right)\right]$$

$$- C_3 C_4 \omega_2 \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] \qquad (17)$$

$$L = \frac{1}{R_4\left[(R_1)^2 + (\omega L_1)^2\right]} \left[R_1 \left(\frac{1}{R_3} + \frac{1}{R_5}\right) + \omega^2 L_1 C_3\right]$$

$$+ \frac{1}{R_3 R_4 R_5} - C_3 C_4 \omega^2 \left[\frac{1}{R_3} + \frac{1}{R_4}\right] \qquad (18)$$

$$M = \frac{\omega}{\left[(R_2)^2 + (\omega L_2)^2\right]} \left[R_2 \left(\frac{1}{R_3} + \frac{1}{R_5}\right) + \omega^2 L_2 C_3\right]$$

$$+ \frac{C_4\omega}{R_4} \left[\frac{1}{R_3} + \frac{1}{R_5}\right] + \frac{\omega C_3}{R_3 R_4} \qquad (19)$$

$$N = \frac{\omega}{R_4\left[(R_1)^2 - (\omega^2 L_1)^2\right]} \left[R_1 C_3 - L_1 \left(\frac{1}{R_3} + \frac{1}{R_5}\right)\right]$$

$$+ \frac{C_4\omega}{R_4} \left[\frac{1}{R_3} + \frac{1}{R_5}\right] + \frac{\omega C_3}{R_3 R_4} \qquad (20)$$

From equations 13 through 20 and equations 7, 11, and 12, the ratio of V_3 to V_4 can be expressed as (assuming $R_1 = R_2$ and $SL_1 = SL_2$):

$$\frac{V_3}{V_4} = \frac{A \cos\theta + B \sin\theta + j\,(-C \cos\theta + D \sin\theta)}{K \cos\theta + L \sin\theta + j\,(+M \cos\theta - N \sin\theta)} \qquad (21)$$

If the resolver source impedances ($R_1 + SL_1$) and ($R_2 + SL_2$) are not equal, equation 21 will take the following form:

$$\frac{V_3}{V_4} = \frac{\dfrac{A}{R_1 + j\omega L_1} \cos\theta + \dfrac{B}{(R_2 + j\omega L_2)} \sin\theta + j\left[\dfrac{-C\cos\theta}{(R_1 + j\omega L_1)} + \dfrac{D \sin\theta}{(R_2 + j\omega L_2)}\right]}{\dfrac{K}{(R_1 + j\omega L_1)} \cos\theta + \dfrac{L \sin\theta}{(R_2 + j\omega L_2)} + j\left[\dfrac{M\cos\theta}{(R_1 + j\omega L_1)} - \dfrac{N \sin\theta}{(R_2 + j\omega L_2)}\right]} \qquad (22)$$

To simplify equation 22, let

$$A' = \frac{R_1}{(Z_1)^2} A - \frac{\omega L_1}{(Z_1)^2} C$$

$$B' = \frac{R_2}{(Z_2)^2} B + \frac{\omega L_2}{(Z_2)^2} D$$

$$C' = \frac{R_1}{(Z_1)^2} C + \frac{\omega L_1}{(Z_1)^2} A$$

$$D' = \frac{R_2}{(Z_2)^2} D - \frac{\omega L_2}{(Z_2)^2} B \qquad (23)$$

$$K' = \frac{R_1}{(Z_1)^2} K + \frac{\omega L_1}{(Z_1)^2} M$$

$$L' = \frac{R_2}{(Z_2)^2} L - \frac{\omega L_2}{(Z_2)^2} N$$

$$M' = \frac{R_1}{(Z_1)^2} M - \frac{\omega L_1}{(Z_1)^2} K$$

$$N' = \frac{R_2}{(Z_2)^2} N + \frac{\omega L_2}{(Z_2)^2} L \qquad (24)$$

where:

$$(Z_1)^2 = (R_1)^2 + (\omega L_1)^2$$

$$(Z_2)^2 = (R_2)^2 + (\omega L_2)^2$$

and each primed number is real.

Equation 22 can now be written:

$$\frac{V_3}{V_4} = \frac{A' \cos\theta + B' \sin\theta + j\left[-C' \cos\theta + D' \sin\theta\right]}{K' \cos\theta + L' \sin\theta + j\left[M' \cos\theta - N' \sin\theta\right]} \qquad (25)$$

Equations 21 and 25 describe ratio of output bridge voltage V_3 and V_4. From these equations (21 ideal case - 25 impedances unequal), it is possible to derive an expression for a phase angle error function. First, consider the numerator of equation 21 or 25, and let:

$$f_1 = A \cos \theta + B \sin \theta + j(-C \cos \theta + D \sin \theta) \tag{26}$$

Make the following substitution:

$$\cos \phi_1 = \frac{A}{(A^2 + B^2)^{\frac{1}{2}}} \quad ; \quad \sin \phi_1 = \frac{B}{(A^2 + B^2)^{\frac{1}{2}}}$$

$$\cos (\phi + \alpha_1) = \frac{D}{(C^2 + D^2)^{\frac{1}{2}}} \quad ; \quad \sin (\phi + \alpha_1) = \frac{C}{(C^2 + D^2)^{\frac{1}{2}}}$$

$$\begin{aligned} f_1 = {} & (A^2 + B^2)^{\frac{1}{2}} (\cos \theta \cos \phi_1 + \sin \theta \sin \phi_1) \\ & + j (C^2 + D^2)^{\frac{1}{2}} \left[\sin \theta \cos (\phi + \alpha_1) - \cos \theta \sin (\phi + \alpha_1)\right] \end{aligned} \tag{27}$$

By trigonometric identities,

$$f_1 = (A^2 + B^2)^{\frac{1}{2}} \cos (\theta - \phi_1) + j (C^2 + D^2)^{\frac{1}{2}} \sin \left[\theta - (\phi_1 + \alpha_1)\right] \tag{28}$$

The phase angle of equation 28 can be written as:

$$(\theta - \phi_1 + \beta_1) = \tan^{-1} \left[\frac{(C^2 + D^2)^{\frac{1}{2}}}{(A^2 + B^2)^{\frac{1}{2}}} \quad \frac{\sin \left[\theta - (\phi_1 + \alpha_1)\right]}{\cos (\theta - \phi_1)} \right] \tag{29}$$

In equation 29, let

$$1 - \epsilon_1 = \frac{(C^2 + D^2)^{\frac{1}{2}}}{(A^2 + B^2)^{\frac{1}{2}}} \tag{30}$$

By use of trigonometric identities and substitution of equation 30, then equation 29 becomes:

$$\frac{\tan (\theta - \phi_1) + \tan \beta_1}{1 - \tan \beta_1 \tan (\theta - \phi_1)} = (1 - \epsilon_1) \left[\tan (\theta - \phi_1) - \tan \alpha_1\right] \tag{31}$$

Assuming that α_1 and β_1 are small angles (results prove the assumption to be correct), equation 31 can be written as:

$$\frac{\tan(\theta - \phi_1) + \beta_1}{1 - \beta_1 \tan(\theta - \phi_1)} = (1 - \epsilon_1) \left[\tan(\theta - \phi_1) - \alpha_1\right] \tag{32}$$

Solving for β_1,

$$\beta_1 = \frac{\frac{1}{2}\left[-\alpha_1 + \alpha_1\epsilon_1 - \alpha_1 \cos 2(\theta - \phi_1) + \alpha_1\epsilon_1 \cos 2(\theta - \phi_1) - \epsilon_1 \sin 2(\theta - \phi_1)\right]}{1 - \frac{1}{2}\alpha_1 \sin 2(\theta - \phi_1) + \epsilon_1 \sin 2(\theta - \phi_1) - \frac{1}{2}\epsilon_1\alpha_1 \sin 2(\theta - \phi_1)} \tag{33}$$

Neglecting second order terms,

$$\beta_1 = \frac{1}{2}\left[-\alpha_1 - \alpha_1 \cos 2(\theta - \phi_1) - \epsilon_1 \sin 2(\theta - \phi_1)\right] \tag{34}$$

Now consider the denominator of equation 21 or 25, and let:

$$f_2 = j\left[-j(K \cos\theta + L \sin\theta) + (M \cos\theta - N \sin\theta)\right] \tag{35}$$

Make the following substitution:

$$\cos(\phi_2 + \alpha_2) = \frac{L}{(K^2 + L^2)^{\frac{1}{2}}} \quad ; \quad \sin(\phi_2 + \alpha_2) = \frac{K}{(K^2 + L^2)^{\frac{1}{2}}}$$

$$\cos(\phi_2) = \frac{M}{(M^2 + N^2)^{\frac{1}{2}}} \quad ; \quad \sin(\phi_2) = \frac{N}{(M^2 + N^2)^{\frac{1}{2}}}$$

and substituting in equation 35

$$f_2 = j\left\{-j(K^2 + L^2)^{\frac{1}{2}}\left[\sin(\phi_2 + \alpha_2)\cos\theta + \cos(\phi_2 + \alpha_2)\sin\theta\right] + (M^2 + N^2)^{\frac{1}{2}}\left[\cos\theta\cos\phi_2 - \sin\theta\sin\phi_2\right]\right\} \tag{36}$$

By trigonometric identities,

$$f_2 = j\left\{(M^2 + N^2)^{\frac{1}{2}}\cos(\theta + \phi_2) - j(K^2 + L^2)^{\frac{1}{2}}\sin\left[\theta + (\phi_2 + \alpha_2)\right]\right\} \tag{37}$$

The phase angle of complex equation 37 can be written as:

$$(\theta + \phi_2 + \beta_2) = \tan^{-1} \left\{ \frac{(K^2 + L^2)^{\frac{1}{2}}}{(M^2 + N^2)^{\frac{1}{2}}} \quad \frac{\sin \left[\theta + (\phi_2 + \alpha_2)\right]}{\cos (\theta + \phi_2)} \right\} \tag{38}$$

In equation 38, let

$$1 - \epsilon_2 = \frac{(K^2 + L^2)^{\frac{1}{2}}}{(M^2 + N^2)^{\frac{1}{2}}} \tag{39}$$

By use of trigonometric identities and substitution of equation 39, then equation 38 can be expressed as:

$$\frac{\tan (\theta + \phi_2) + \tan \beta_2}{1 - \tan \beta_1 \tan (\theta + \phi_2)} = (1 - \epsilon_2) \left[\tan (\theta + \phi_2) + \tan \alpha_2\right] \tag{40}$$

Assuming α_2 and β_2 are small angles, equation 40 can be written as:

$$\frac{\tan (\theta + \phi_2) + \beta_2}{1 - \beta_2 \tan (\theta + \phi_2)} = (1 - \epsilon_2) \left[\tan (\theta + \phi_2) + \alpha_2\right] \tag{41}$$

Solving for β_2,

$$\beta_2 = \frac{\frac{1}{2} \left[\alpha_2 + \alpha_2 \cos 2 (\theta + \phi_2) - \epsilon_2 \alpha_2 - \epsilon_2 - \alpha_2 \cos 2 (\theta + \phi_2) - \epsilon_2 \sin 2(\theta + \phi_2)\right]}{1 + \frac{\alpha_2}{2} \sin 2(\theta + \phi_2) - \epsilon_2 \sin 2 (\theta + \phi_2) - \frac{\epsilon_2 \alpha_2}{2} \sin 2 (\theta + \phi_2)} \tag{42}$$

Neglecting second order terms, β_2 can be written as:

$$\beta_2 = \frac{1}{2} \left[\alpha_2 + \alpha_2 \cos 2 (\theta + \phi_2) - \epsilon_2 \sin 2 (\theta + \phi_2)\right] \tag{43}$$

From equations 6, 29, and 38, the phase angle of V_3 with respect to V_4 can be expressed as:

$$\zeta = (2\theta - \frac{\pi}{2}) + (-\phi_1 + \beta_1) + (\phi_2 + \beta_2) \tag{44}$$

Since equation 6 was derived for an ideal resolver, the phase angle error function is:

$$\epsilon_{\zeta} = \phi_2 - \phi_1 + \beta_1 + \beta_2 \tag{45}$$

The expressions of β_1 and β_2 have been derived and are given as equations 34 and 43. Making these substitutions in equation 45, the error function can be expressed as:

$$\epsilon_{\zeta} = \phi_2 - \phi_1 + \frac{1}{2}\left[\alpha_2 - \alpha_1\right] + \frac{\alpha_2}{2}\cos 2(\theta + \phi_2) - \frac{\epsilon_2}{2}\sin 2(\theta + \phi_2) - \frac{\alpha_1}{2}\cos 2(\theta - \phi_1) - \frac{\epsilon_1}{2}\sin 2(\theta - \phi_1) \tag{46}$$

A two-cycle error as well as a constant bias error will be generated from an imperfect resolver. If the network schematic shown in FIGURE 5 can be set so that

$$(R_1 + SL_1) = (R_2 + SL_2)$$

$$R_3 = R_4 = \frac{1}{\omega C_3} = \frac{1}{\omega C_4}$$

$$R_5 = R_6$$

then the bias term is approximately zero or

$$\phi_2 - \phi_1 + \frac{1}{2}(\alpha_2 - \alpha_1) \approx 0 \tag{47}$$

The error function previously derived and as defined in equation 46 has been evaluated and the nominal error computed. The following set of specifications has been established from preliminary design computation and was used in the error computations.

Number of poles - 64

Primary winding - rotor

Primary voltage - 26 volts - 2000 Hz

Primary phases - 2 (one short-circuited)

Secondary phases - 2

Primary current - 0.09 amps

Primary power - 0.6 watts

Primary impedance - 70 + j 280 ohms (secondary open)

Secondary impedance - 15 + j 45 ohms (primary short-circuited)

Voltage ratio - 0.192

Secondary phase shift - 12 degrees (open-circuited)

Secondary voltage (maximum) - 5.0 volts (open-circuited)

D.c. resistance primary - 68 ohms

D.c. resistance secondary - 11 ohms

Manufacturing tolerance secondary impedance - 15 ± 10 per cent + j 45 ± 10 per cent ohms

Mechanical accuracy - ± 10 arc sec

The nominal error is shown in FIGURE 6.

$$\text{Nominal error} = \pm \frac{640}{64} = \pm 10 \text{ arc sec}$$

Bias = 0

A perturbation study of the different elements with the following nominal values and variations has been made.

Nominal Bridge Elements

$$R_3 = R_4 = 20\ 000 \text{ ohms}$$

$$\frac{1}{\omega C_3} = \frac{1}{\omega C_4} = 20\ 000 \text{ ohms}$$

Bridge Match

$$R_3 = \frac{1}{\omega C_3} = 20\ 000 \pm 1 \text{ per cent ohms}$$

$$R_4 = \frac{1}{\omega C_4} = 20\ 000 \pm 1 \text{ per cent ohms}$$

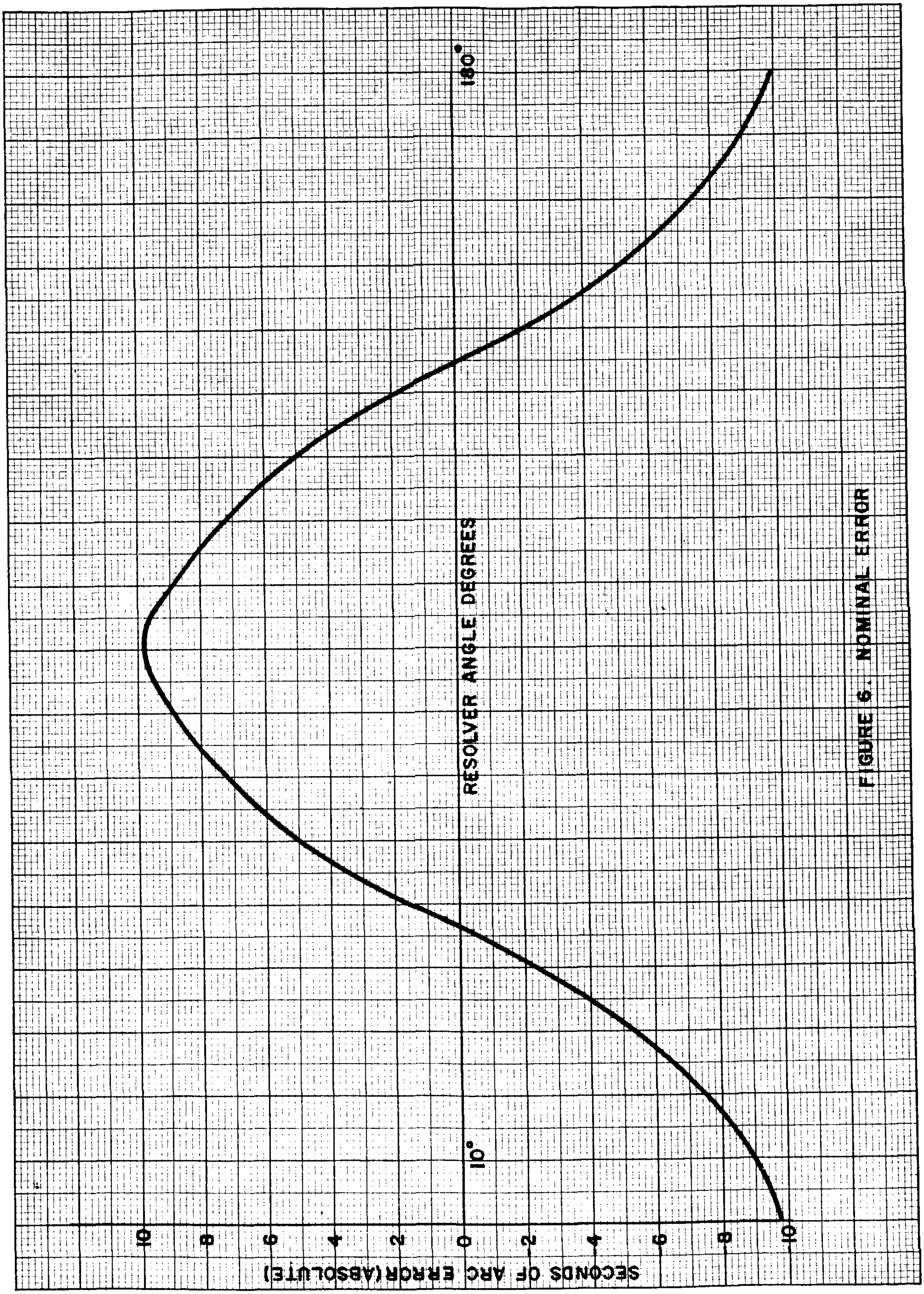


FIGURE 6. NOMINAL ERROR

This match means that the condenser would have a tolerance of ± 1 per cent, and the resistor would be perfectly trimmed against the selected condenser.

Bridge Element Variation

$\Delta R_3 = \pm \frac{1}{4}$ per cent from nominal value

$\Delta \frac{1}{\omega C_3} = \pm \frac{1}{4}$ per cent from nominal value

$\Delta \frac{1}{\omega C_4} = \pm \frac{1}{4}$ per cent from nominal value

Slipring resistance variation in each stator lead - 0 to 5 ohms.

Platform ambient variation - ± 50 degrees C

Nominal Load Resistance

$R_5 = R_6 = 100\ 000$ ohms

Load Resistance Variation

$\Delta R_5 = \pm\ 1000$ ohms

$\Delta R_6 = \pm\ 1000$ ohms

All errors introduced as a result of these tolerances are random type errors; thus the root sum square is a measure of the three sigma accuracy of the resolver network. A plot of the root sum square error is presented in FIGURE 7. The nominal error shown in FIGURE 6 is not statistical; thus the expected error must be plotted as a random deviation from the nominal and is presented in FIGURE 8. (Note: all curves are absolute values.) Table I is a tabulation of the basic error and maximum cyclic errors for the previously defined tolerances.

An additional error source, which must be considered, is shaft angle error obtained from harmonics generated within the resolver.

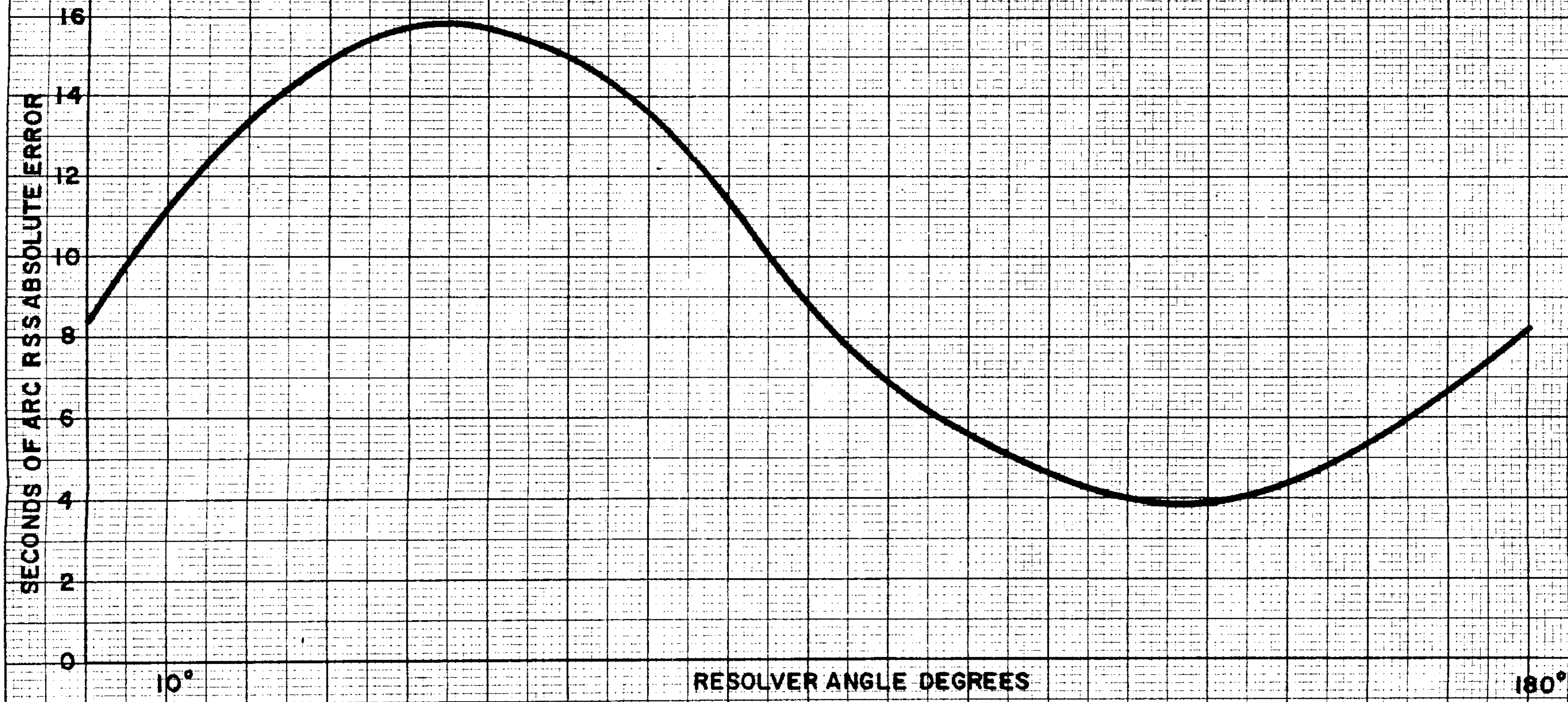


FIGURE 7. RSS ERROR (WITH SPECIFICATION TOLERANCES) VERSUS RESOLVER ANGLE

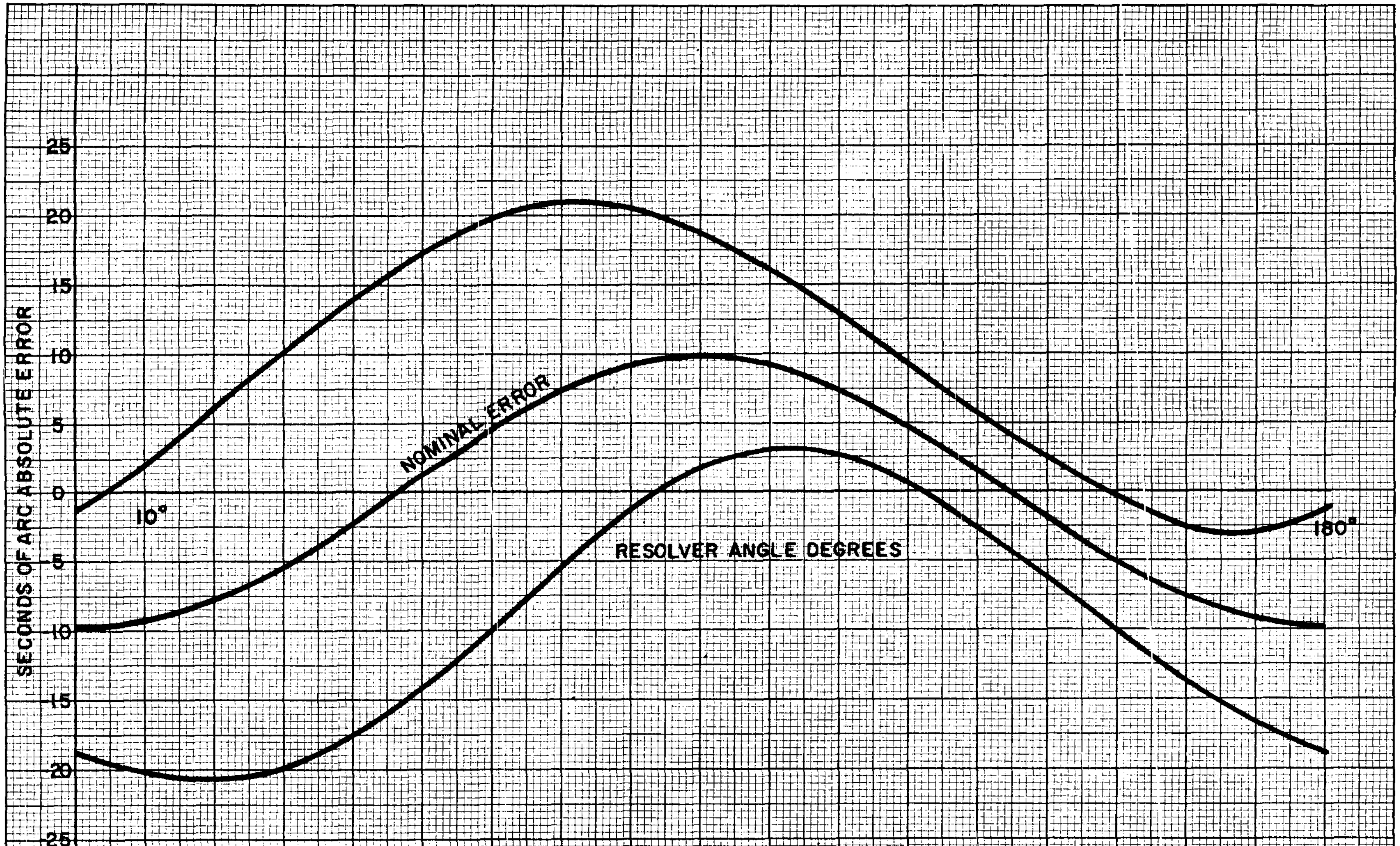


FIGURE 8. RANDOM DEVIATION FROM NOMINAL ERROR VERSUS RESOLVER ANGLE

TABLE I

	Tolerance (ohms)	Bias arc sec	Peak cyclic error arc sec	Type
scheme	nominal	0	638	nominal error
slipring	$\Delta R_1 = +5$	-42	6.6	incremental error
slipring	$\Delta R_1 = -5$	+42	6.6	incremental error
slipring	$\Delta R_2 = +5$	+42	6.6	incremental error
slipring	$\Delta R_2 = -5$	-42	6.6	incremental error
temperature	$\Delta R_1 = \Delta R_2 = +5$	0	85	incremental error
temperature	$\Delta R_1 = \Delta R_2 = -5$	0	85	incremental error
manufacturing	$\Delta R_1 = +1.5$	-21	33	incremental error
manufacturing	$\Delta R_1 = -1.5$	+21	33	incremental error
manufacturing	$\Delta \omega L_1 = +4.5$	+53	74	incremental error
manufacturing	$\Delta \omega L_1 = -4.5$	-53	74	incremental error
manufacturing	$\Delta R_2 = +1.5$	+21	33	incremental error
manufacturing	$\Delta R_2 = -1.5$	-21	33	incremental error
manufacturing	$\Delta \omega L_2 = +4.5$	-53	74	incremental error
manufacturing	$\Delta \omega L_2 = -4.5$	+53	74	incremental error
bridge match	$\Delta R_3 = \Delta \frac{1}{\omega C_3} = +50$	+45	3	incremental error
bridge match	$\Delta R_3 = \Delta \frac{1}{\omega C_3} = -50$	-40	3	incremental error
bridge match	$\Delta R_4 = \Delta \frac{1}{\omega C_4} = +50$	-45	3	incremental error
bridge match	$\Delta R_4 = \Delta \frac{1}{\omega C_4} = -50$	+40	3	incremental error
bridge variation	$\Delta R_3 = +50$	-210	253	incremental error
bridge variation	$\Delta R_3 = -50$	+212	258	incremental error
bridge variation	$\Delta \frac{1}{\omega C_3} = +50$	+254	258	incremental error

Note: all absolute error values are 1/64 of tabulated value.

TABLE I (Cont'd)

	Tolerance (ohms)	Bias arc sec	Peak cyclic error arc sec	Type
bridge variation	$\Delta\frac{1}{\omega C_3} = -50$	-252	253	incremental error
bridge variation	$\Delta R_4 = +50$	+210	253	incremental error
bridge variation	$\Delta R_4 = -50$	-212	253	incremental error
bridge variation	$\Delta\frac{1}{\omega C_4} = +50$	-254	258	incremental error
bridge variation	$\Delta\frac{1}{\omega C_4} = -50$	+210	253	incremental error
load variation	$\Delta R_5 = +1000$	-155	0	incremental error
load variation	$\Delta R_5 = -1000$	+186	1	incremental error
load variation	$\Delta R_6 = +1000$	+155	0	incremental error
load variation	$\Delta R_6 = -1000$	-186	1	incremental error

Note: all absolute error values are 1/64 of tabulated value.

Assume that the resolver stator contains third and fifth harmonic components. The worst possible physical location of these harmonic generators is shown in FIGURE 9.

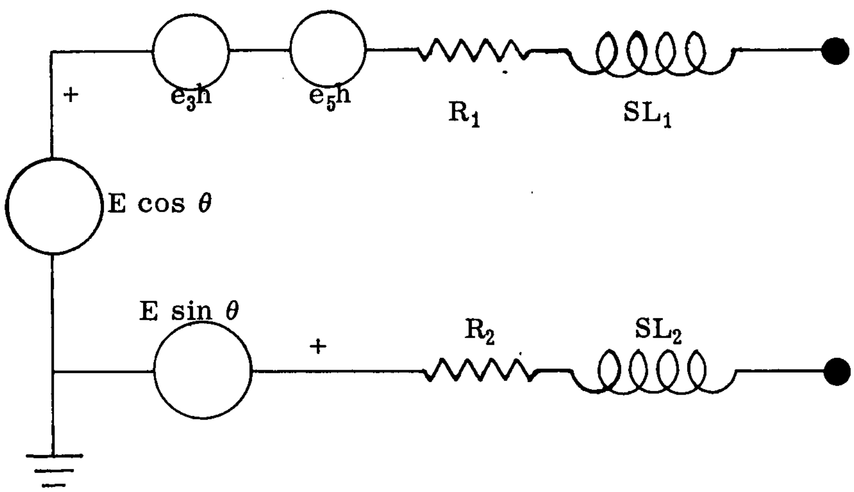


FIGURE 9. HARMONIC GENERATION

For this investigation, assume the resolver has zero source impedance $(R_1 + SL_1) = (R_2 + SL_2) = 0$ and the load resistors are infinite $(R_5 = R_6 = \infty)$. Any harmonic voltage generated would be independent of shaft rotation and would have a constant phase angle reference with respect to V_3 (t) when $\theta = 45$ degrees.

First consider the third harmonic. Let the magnitude be written as:

$$E_{3rd} = \eta_3 E \tag{48}$$

From equations 7 and 48, the instantaneous voltages $V_3(t)$ and $V_4(t)$ must now be expressed as:

$$V_3(t) = \frac{E}{\sqrt{2}} \sin\left(\omega t + \theta - \frac{\pi}{4}\right) + 0.316\, \eta_3 E \sin(3\omega t + \delta_3)$$

$$V_4(t) = \frac{E}{\sqrt{2}} \sin\left(\omega t - \theta + \frac{\pi}{4}\right) + 0.95\, \eta_3 E \sin(3\omega t + \delta_3) \tag{49}$$

where δ_3 is an arbitrary phase angle.

If there is no harmonic content in the resolver output, V_3 will generate a start pulse when

$$\omega t_3 = -\theta + \frac{\pi}{4}$$

and V_4 will generate a stop pulse when

$$\omega t_4 = \theta - \frac{\pi}{4}$$

The harmonic terms in $V_3(t)$ and $V_4(t)$ expressed in equation 49 will cause ωt_3 and ωt_4 to deviate by angles ϵ_3 and ϵ_4, respectively. Thus, from equation 49

$$\epsilon_3 = .447\ \eta_3 \sin\left(3\theta - \delta_3 - \frac{3\pi}{4}\right)$$

$$\epsilon_4 = -1.34\ \eta_3 \cos\left(3\theta + \delta_3 - \frac{3\pi}{4}\right) \qquad (50)$$

The phase angle error can be expressed as:

$$\begin{aligned} \epsilon_{\gamma_3} &= \epsilon_4 - \epsilon_3 \\ &= -\eta_3 \left\{\cos\left(3\theta - \frac{3\pi}{4}\right)\left[1.34 \cos\delta_3 + .447 \sin\delta_3\right]\right. \\ &\quad \left. - \sin\left(3\theta - \frac{3\pi}{4}\right)\left[1.34 \sin\delta_3 - .447 \cos\delta_3\right]\right\} \end{aligned} \qquad (51)$$

which can be simplified:

$$\epsilon_{\gamma_3} = -1.41\ \eta_3 \cos\left(3\theta - \frac{3\pi}{4} + \delta_3'\right) \qquad (52)$$

where

$$\delta_3' = \cos\delta_3' = \left[\frac{1.34 \cos\delta_3 + .447 \sin\delta_3}{\sqrt{2}}\right]$$

The fifth harmonic error can be derived with the same procedure as above and can be expressed:

$$\epsilon_{\gamma_5} = -1.46\ \eta_5 \cos\left(5\theta - \frac{5\pi}{4} + \delta_5'\right) \qquad (53)$$

Assuming that the third and fifth harmonic can contribute a maximum of 5 arc sec of shaft error, then:

$$\epsilon_{\gamma_3} = \epsilon_{\gamma_5} = 1.55 \times 10^{-3} \text{ radians} \qquad (54)$$

or

$$\eta_3 = \eta_5 = 1.09 \times 10^{-3} \qquad (55)$$

Thus, the third and fifth harmonic content of the resolver must be held to less than 0.1 per cent. Harmonic content is greatly increased if the resolver is operated in any condition of core saturation. Therefore, core design is based on a 52 volts RMS saturation, and the highest quality magnetic material is used.

The resolver is a 64 pole or 32 electrical speed unit; assume that the mechanical shaft angle is represented by Ω. Then, the electrical phase angle of V_3/V_4 is:

$$\gamma = 64\,\Omega + \epsilon\,(\phi) \qquad (56)$$

where $\epsilon\,(\phi)$ is the total error function. The number of cycles to be counted for any given value of γ should be the exact number of arc seconds in Ω. The number of cycles to be counted can be expressed as:

$$N = \frac{\gamma}{2\pi} \quad \frac{f_C}{f_R} \qquad (57)$$

where

$f_C \triangleq$ clock frequency

$f_R \triangleq$ resolver carrier frequency 2000 Hz

Assume $\gamma = 2\pi$ (360 degrees); then Ω is exactly $\frac{360 \text{ degrees}}{64}$ or 5.625 degrees or 20 250 arc sec. If a requirement exists to count each arc sec, then:

$$f_C = N\,f_R = 20\ 250 \times 2000 = 40.50 \times 10^6 \text{ Hz} \qquad (58)$$

An accuracy of 0.01 per cent for both f_C and f_R would yield a shaft error of $\pm$ 2 arc sec for each frequency.

If the clock frequency is 2×10^6 Hz and the resolver excitation frequency is 2000 Hz, the count would be 20.25 arc sec.

Decreasing the resolver excitation frequency to 1000 Hz would half the count to 10.125 arc sec. The excitation frequency should be held as high as possible to reduce harmonic errors and minimize power consumption.

The shaft angle encoder is a static device; thus if it is used to measure angles in a dynamic condition, the measurement will be in error. If a 10 arc sec accuracy of the measurement is desired under dynamic conditions, the maximum angular rate should be restricted to a maximum of 0.2 rad/s.

CONCLUSION

The shaft angle resolver encoder will have:

a. Mechanical accuracy of ± 10 arc sec.

b. An electrical accuracy of ± 21.2 arc sec because of tolerances and perturbation.

c. A distortion accuracy of ± 5 arc sec for a ± 0.1 per cent third harmonic.

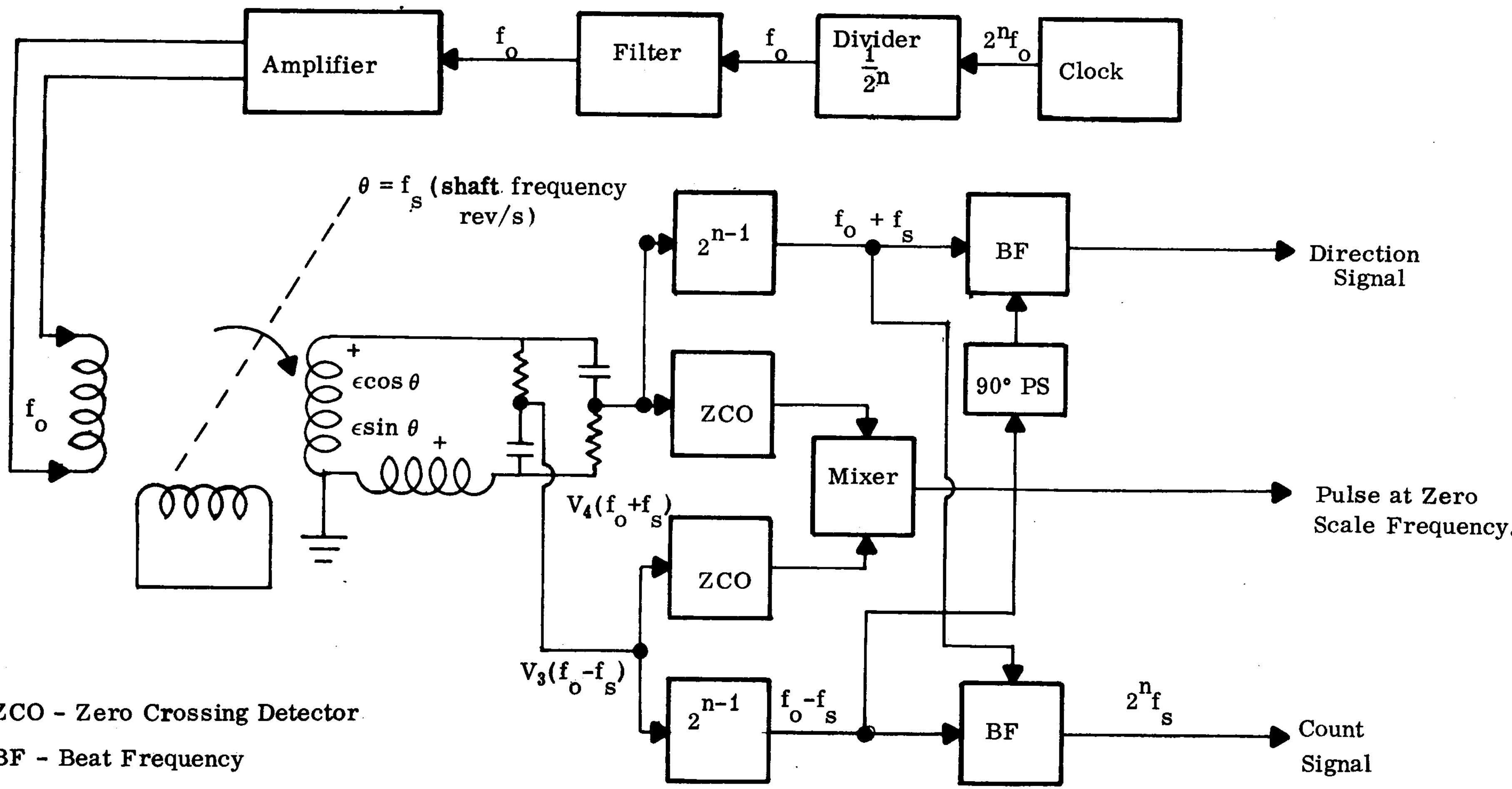
d. A distortion accuracy of ± 5 arc sec for a ± 0.1 per cent fifth harmonic.

e. An accuracy of ± 2 arc sec for a resolver carrier frequency tolerance of ± 0.01 per cent and ± 2 arc sec for a clock frequency tolerance of ± 0.01 per cent.

f. Maximum angular rate limited to 0.2 rad/s.

g. A clock frequency of 2×10^6 Hz and 2000 Hz excitation frequency will have an incremented count of 20.25 arc sec.

A simplified block diagram of the double-angle digital encoder is shown in FIGURE 10. The single speed section is not shown but operates in the same manner as the multi-speed, except single angle encoding will be used. The single output is to generate an integral number which represents the number of poles that the 32 speed resolver has been rotated from its reference position.



ZCO - Zero Crossing Detector

BF - Beat Frequency

PS - Phase Shifter

FIGURE 10. DOUBLE ANGLE ENCODER BLOCK DIAGRAM.

MTP-ASTR-G-63-7

APPROVAL

MULTI-SPEED RESOLVERS FOR ANALOG DIGITAL CONVERSION OF SHAFT ANGLES

By

H. E. Thomason

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has been reviewed for technical accuracy.

Carl H. Mandel

CARL H. MANDEL
Chief, Gyro and Stabilizer Branch

W. Haeussermann

W. HAEUSSERMANN
Director, Astrionics Division

DISTRIBUTION

M-DIR

M-SAT
Dr. Lange
Mr. Powell

M-QUAL
Mr. Grau
Mr. C. Brooks
Mr. P. Hall
Mr. L. Hall

M-LOV
Dr. Gruene
Mr. Chambers (3)
Mr. Jenke

M-AERO
Dr. Geissler
Dr. Spear
Mr. Linberg

M-ME
Mr. Kuers

M-TEST
Mr. Heimburg

M-P&VE
Mr. Mrazek

M-ASTR
Dr. Haeussermann
Mr. Brandner
Mr. Digesu
Mr. Fichtner
Mr. Richard (12)
Mr. Weber
Mr. J. Taylor
Mr. F. B. Moore
Mr. C. Swearingen
Mr. J. Bridges
Mr. J. Mack
Mr. Mandel
Mr. Thomason (18)
Miss Flowers
Reference File
Branch File

M-MS-IP

M-MS-IPL (8)

M-PAT

M-MS-H

M-HME-P

Scientific and Information Facility (2)
Attn: NASA Representative (S-AK/RKT)
P. O. Box 5700
Bethesda, Maryland